WAVELET USE IN BIOMEDICAL IMAGE DENOISING

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Abstract

Algorithms of image denoising form a fundamental tool in many problems related to engineering, environmental and biomedical systems. The paper presents the possibility of wavelet use in signal decomposition and reconstruction using selected method of thresholding to suppress undesirable signal components. Applications studied in the paper include biomedical signal and image processing. Resulting algorithms form a system allowing remote data processing based on the use of the Matlab Web Server.

1 Introduction

Image denoising and enhancement represent fundamental problems of image processing in various information systems with applications including biomedical and engineering signals. Problems closely related to this topic include signal restoration, statistical signal processing, image segmentation and classification. Data acquisition forms the initial part of signal and image processing followed by feature extraction, dimensionality reduction and compression [1] in many cases. An example of a biomedical signal standing for a NMR brain image is presented in Fig. 1 followed by contours of this image before and after its denoising given in Figs. 2 and 3.

The main part of the paper is devoted to basic principles of Wavelet transform and its application in image analysis, its decomposition and reconstruction and additive noise reduction. The emphasis is in wavelet choice and their use in this area. Results of such an image denoising are compared with that achieved by FIR filters.

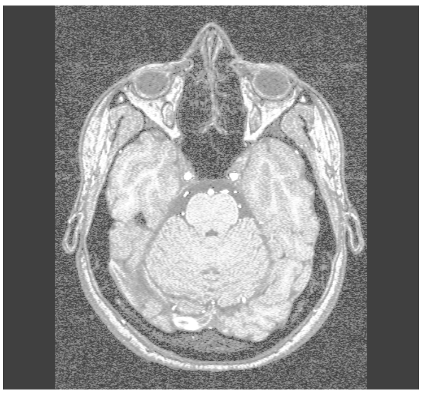

Figure 1: Original NMR image

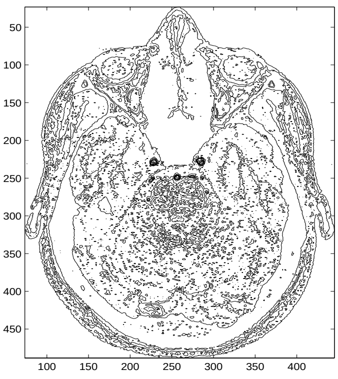

Figure 2: Contour plot of the original image

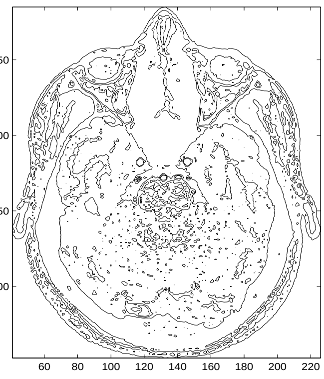

Figure 3: Contour plot of the final image

2 Principles of Image Wavelet Decomposition

Wavelet transform (WT) provide the alternative to the short-time Fourier transform (STFT) for non-stationary signal analysis [5, 3]. Both STFT and WT result in signal decomposition into two-dimensional function of time and frequency respectively scale that is closely related to frequency in some cases. The basic difference between these two transforms is in the construction of the window function which has a constant length in case of the STFT (including rectangular, Blackman and other window functions) while in case of the WT wide windows are applied for

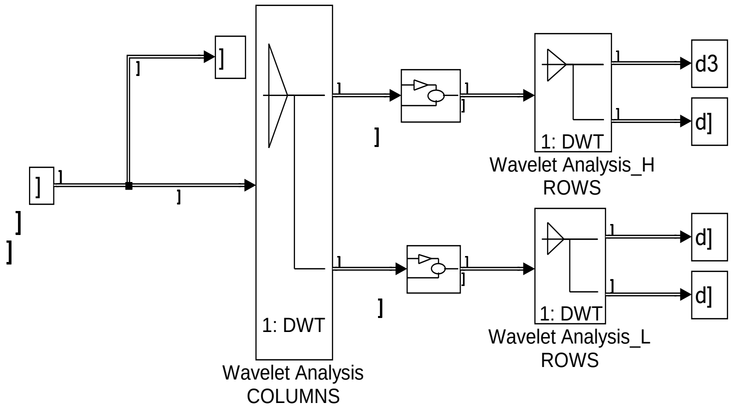

Figure 4: The fundamental part of wavelet tree for image decomposition and subsampling in the Simulink environment

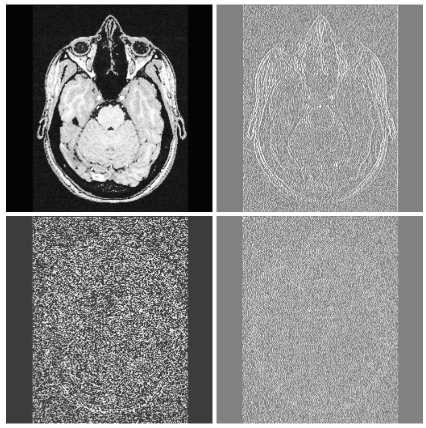

Figure 5: The first stage of image decomposition presenting images **d0**, **d1**, **d2** and **d3**

low frequencies and short windows for high frequencies to enable varying time and frequency resolutions. Local and global signal analysis can be combined in this way.

Wavelet functions used for signal analysis are derived from the initial basic (mother) function $W(t)$ forming the set of functions

$$W_{m,k}(t) = \frac{1}{\sqrt{a}}\, W\, (\frac{1}{a}\, (t-b)) = \frac{1}{\sqrt{2^m}}\, W\, (2^{-m}t - k)) \tag{1}$$

for discrete parameters of dilation $a = 2^m$ and translation $b = k\ 2^m$.

The basic efficient way to evaluate wavelet transform coefficients using the signal processing notation assumes implementation of the Mallat's pyramidal structure of wavelet transform coefficients. In case of decomposition of images it is possible to use a tree structure [4] presented for one step of image decomposition in Fig. 4 using Simulink environment and its notation. This algorithm assumes the use of the half band low-pass scaling sequence $\{l(n)\}_{n=0}^{L-1}$ together with the corresponding wavelet sequence $\{h(n)\}_{n=0}^{L-1}$ and their convolution with the analyzed image $\mathbf{A}$ in image columns and then rows subsampling results by two in both cases. The image is decomposed in this way into 4 subimages presented in Fig. 5 for the biomedical image obtained by the nuclear magnetic resonance (NMR).

3 Wavelet Noise Reduction

The denoising algorithm assumes that the signal has low frequency components and that it is corrupted by the additive Gaussian noise with its power much lower than that of the analyzed signal. The whole method consists of the following steps presented in Fig. 6 for a chosen segment of EEG signal:

- Signal decomposition using a selected wavelet function up to the given level and evaluation of wavelet transform coefficients
- The choice of threshold limits for each decomposition level and modification of its coefficients
- Signal reconstruction from modified wavelet transform coefficients

Results of this process depend on the proper choice of wavelet functions and the selection of threshold limits.

The application of threshold limits to modify wavelet coefficients $\{c(k)\}_{k=0}^{N-1}$ results in the case of soft thresholding and a chosen thresholding value δ in the evaluation of new coefficients

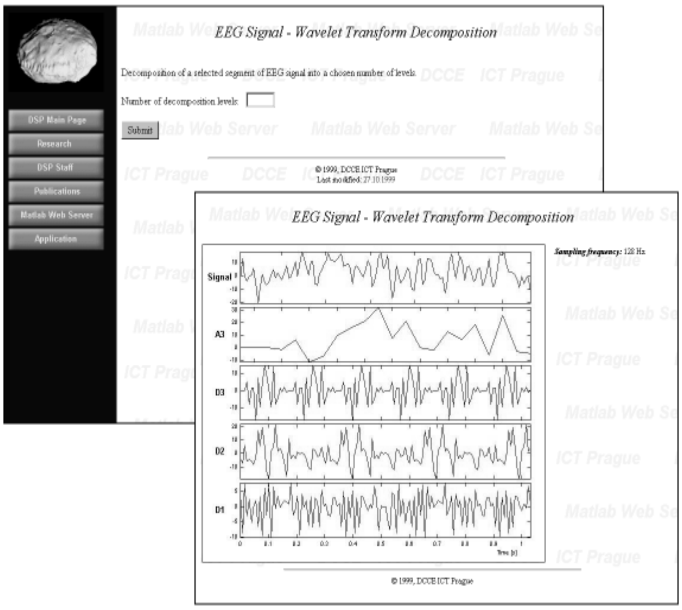

Figure 6: DSP Matlab server and its use for remote EEG data decomposition into a selected number of levels

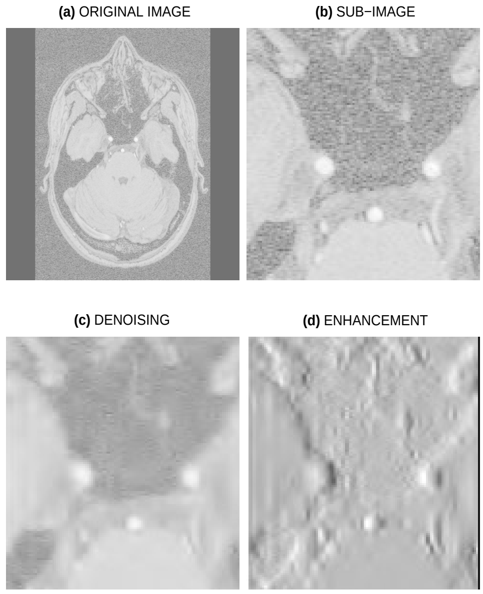

Figure 7: Results of Wavelet decomposition use for NMR image denoising followed by its gradient enhancement

$$\overline{c}_s(k) = \begin{cases} \text{sign } c(k) \; (|\, c(k)\,| - \delta) & \text{if } |\, c(k)\,| > \delta \\ 0 & \text{if } |\, c(k)\,| \leq \delta \end{cases} \tag{2}$$

that are used for the following signal reconstruction.

Results of such a process applied to a selected NMR image are presented in Fig. 7. Wavelet denoising applied to a selected subimage is able to reduce the noise and the following gradient signal enhancement significantly improves the structure of the whole image.

4 FIR Filters in Image Denoising

Application of FIR filters is another way parallel to the use of the wavelet transform decomposition which can be useful in noise removal from an image or other data. Denoising of data by applying a FIR filter is an alternative method that has been used for biomedical signal analysis.

As we work with two-dimensional data $x(m,n)$ for $m = 0, \ldots, K-1$, and $n = 0, \ldots, L-1$, the filter we apply must be two-dimensional as well. We can assume its output in the following form

$$y(m,n) = \sum_{k=0}^{K-1} \sum_{l=0}^{L-1} h(k,l) x(m-k, n-l) \tag{3}$$

where $h(k,l)$ are the filter coefficients. Assume a given two-dimensional signal containing two frequency components ω_1, ω_2

$$x(m,n) = e^{j\omega_1 m} e^{j\omega_2 n} \tag{4}$$

and the desired frequency response $e^{-j\,\omega_1 \alpha_1} e^{-j\,\omega_2 \alpha_2}$ for $-\omega_{1c} < \omega_1 < \omega_{1c}$ and $-\omega_{2c} < \omega_2 < \omega_{2c}$, and 0 elsewhere. Then we obtain the filter coefficients by applying the formula for the two-dimensional Fourier series, and we find the following solution for the coefficients

$$h(m,n) = \frac{\sin \omega_{1c}(m-\alpha_1)}{m-\alpha_1} \; \frac{\sin \omega_{2c}(n-\alpha_2)}{n-\alpha_2} \tag{5}$$

where $\alpha_1 = \frac{M-1}{2}$ and $\alpha_2 = \frac{N-1}{2}$. This filter has been applied to simulated as well as real data, and other filters have been used which are incorporated in the MATLAB program, and whose description can be found in related MATLAB manuals. Detailed explanation of this filter presented here can be found in [2].

In Fig. 8, a simulated two dimensional signal is shown with added noise, its spectrum together with frequency characteristics of FIR filter and then the same image denoised using proposed FIR filter. Similar method has been used for real data processing [2].

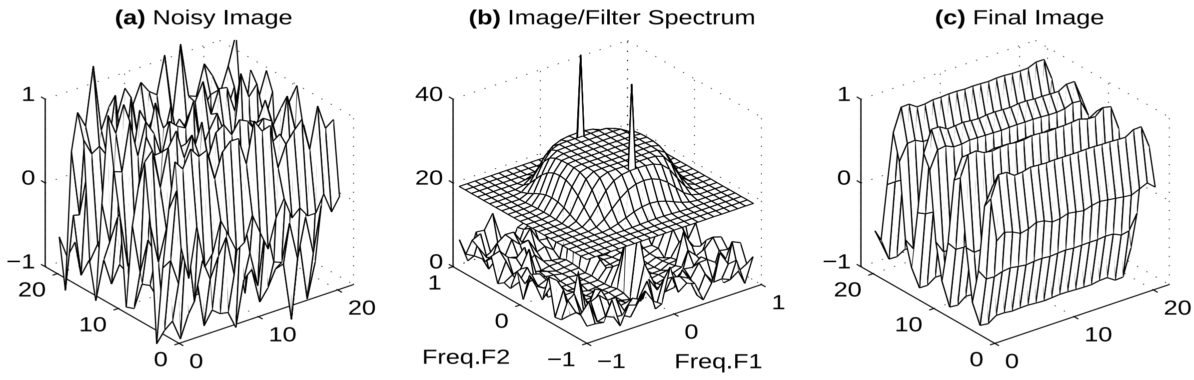

Figure 8: (a) Simulated signal, (b) spectrum of simulated image and FIR filter, (c) final image

5 Conclusion

The paper presents principles of signal decomposition and reconstruction by two dimensional Wavelet transform and compares its results with FIR filtering. Basic results of the contribution are in (i) design and verification of proposed algorithms and in (ii) the introduction of a general Matlab Web server allowing remote data analysis and processing with the example of its use for EEG signal decomposition in Fig. 6. General mathematical principles of image proceesing presented in the paper were applied to biomedical image enhancement presented in Fig. 7.

References

[1] S. Haykin. *Neural Networks.* IEEE Press, New York, 1994.

[2] M. Kolínová and A. Procházka. Two-Dimensional Fourier Transform in Image Noise Rejection. In *Process Control '99*, Tatransk Matliare, 1999. Slovak University of Technology.

[3] D. E. Newland. *An Introduction to Random Vibrations, Spectral and Wavelet Analysis.* Longman Scientific & Technical, Essex, U.K., third edition, 1994.

[4] N.G. Kingsbury and J.F.A. Mugarey. Wavelet Transforms in Image Processing. In A. Procházka, J. Uhlíř, P. J. W. Rayner, and N. G. Kingsbury, editors, *Signal Analysis and Prediction*, Applied and Numerical Harmonic Analysis, chapter 2. Birkhauser, Boston, U.S.A., 1998.

[5] G. Strang. Wavelets and Dilation Equations: A brief introduction. *SIAM Review*, 31(4):614–627, December 1989.

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